Cartels and Search

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Abstract

This paper unifies two significant but somewhat contradictory ideas. First, search costs potentially influence market price equilibria significantly; in many equilibria consumers do not search despite above-competitive prices. Second, cartels must guard against individual members offering lower prices, thereby creating incentives for consumers to search. We develop a simple framework, and then an example, in which whether search takes place depends upon the magnitude of search costs. Three potential equilibria result, dependent upon model parameters. These include a tacit cartel agreement exhibiting price variance and volatility. A policy conclusion is that such market characteristics do not always guarantee non-cartelisation.

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1. Introduction

The cartel literature and the search cost literature reach strikingly different conclusions. Traditionally, one of the key problems discussed in relation to cartels is that of dissuading firms from deviating from an agreed upon price (see e.g. Vives, 1999, chapter 9). However, within the search literature, the classic model of Diamond (1971) is commonly taken to show that so long as search costs, however small, exist, firms independently arrive at the monopoly price without the need for coordination. Anderson and Renault (1999, pp. 719-20) put it as follows: "Suppose there were an equilibrium at which some firm set a price below the monopoly price and (weakly) below those set by all other firms. Then all consumers who go to the low-price firm would still buy there even if the firm raised its price by an amount less than the cost of searching another firm, since a consumer who searched again could not expect to gain enough to offset the search cost. This incentive to raise price means that *all firms must charge the monopoly price in equilibrium*." (emphasis added). Our aim in this paper is to provide a unified treatment of cartel behaviour under conditions of non-zero search costs.

Fundamentally, firms would prefer that consumers remain their customers because they perceive no advantage in purchasing elsewhere. Firms will not lower price below the monopoly level if they think they will gain insufficient additional customers (revenue) to make the price cut worthwhile. This may either be because other firms would follow suit in lowering prices (or threaten to do so as a punishment) or because consumers find it not worthwhile to search.

If search costs exist, consumers may yet search if they anticipate substantial benefits from doing so. Thus what is misleading about the Anderson and Renault argument is that it holds only for a small change in price. If consumers expect a large gain from search, significant numbers may search and hence the equilibrium in which all firms set monopoly price can be broken. Thus allowing for consumer expectations, the Diamond equilibrium survives only in a subset of cases, those where it is not profitable for a single firm to reduce price sufficiently to provoke search behaviour among consumers and hence higher sales. Here we look for equilibria, taking into account both search costs and punishments.

This new approach significantly changes perspectives on cartel behaviour. Specifically, we identify three different types of equilibrium. Only one of them is of the traditional cartel type, that is monopoly pricing supported by a cartel of firms, with punishment for any firm that cheats. The second is monopoly pricing supported by search costs alone- the "Diamond equilibrium". The third is the most novel- where only a lower than monopoly price is supported by search costs and punishments. In this last case, there are at least two prices set in the industry. Thus, we see that coordination need not mean setting the same price levels even in equilibrium in a completely symmetric model. This final possibility provides a considerable challenge for policy purposes, since it cannot be identified through traditional means such as parallel industry-wide movements in prices.

This paper uses a simply-structured model to illustrate the issues involved. After setting out the framework somewhat more generally, for the most part, we consider an example where there are three identical firms serving a measure N of consumers.

Three firms allow us to capture the essential elements of the story whilst avoiding the complications of a myriad of cases. Consumers buy at the cheapest price of which they are aware, providing it is less than their valuation, m. Firms' costs are ignored. One fundamental parameter is the level of search costs, s; clearly s can range between zero and m. The second is the (discounted) period over which cartel punishment is administered, τ.

Section 2 sets out the general framework we use. In section 3, we employ a simple three firm version that is sufficient to capture the issues involved but tractable enough for the outcomes to be illustrated in a diagram. Section 4 explores in a simple manner the impact of introducing a group of consumers who have a lower cost of search, for example as a result of carrying out an internet search. A number of policy issues can be considered within this framework. We consider two, a transparency remedy for cartel behaviour and a "state-specific" strategy, explained later in sections 5 and 6. Section 7 considers entry and 8 concludes.

2. The *n*-firm modelling framework

We proceed below first by setting out a general framework with an arbitrary fixed number of firms. We then develop an extended analysis using a 3-firm example in which we examine the possible range of equilibria.

There are n firms and a measure N of consumers. There are no variable costs. The n firms are all members of the cartel. The cartel may be an explicit cartel or an implicit informal arrangement. The N consumers may vary in identities from one period to another, but the total number is assumed constant. An individual becomes a consumer if she sees a notice or advertisement or the actual product within a store, and the price is set to no more than m. Each consumer is then aware of the product and is also aware of the price at which it can be purchased from the seller. The distribution of this initial product awareness is assumed to be symmetric and so each firm has N/n such consumers. Each consumer buys at most one unit on product. There are numerous other locations where the product might be found for sale. Thus, at this stage consumers may simply decide to purchase the product they have seen, or they might decide to explore the market to see if another seller has the same or similar product at a lower price.

We assume this decision is facilitated by agents who collect product availability and prices from all possible sellers. The agents might be commercial organisations or they night be informal networks of consumers. In the former case, the minimum price (or the best deal) could be announced and the location of this price be notified to a consumer at a cost of s. Alternatively, the consumer could explore her network of contacts and then arrange or commission the purchase of the best deal for a cost of s. In both scenarios, the best deal is known and the cost s reveals to the consumer where the best deal is to be found and covers any extra transaction costs. The cost of purchasing the search, s, is termed the search cost. It is implemented if the known gain from search (the improvement in the deal relative to the observed product) is higher than s.

In the set-up outlined above consumers have no uncertainty. Either they buy where they know the product is available or they rationally spend s for a gain of more than s and buy the best deal. In this situation, a one-shot game is very simple. Each firm sets a price $p \leq m$. One possible Nash equilibrium in pure strategies is for each firm to set price at the monopoly price m. No firm would wish to deviate from this if the profit mN/n is no less than the profit from cutting price to m-s and selling N units, that is one to each of the N consumers in the market: i.e. if

$$mN/n \geq (m-s)N$$

Thus for a monopoly equilibrium to exist without the need for coordination requires:[1]

$$s \geq (1-1/n)m \tag{1}$$

This condition makes it difficult for a "Diamond equilibrium" to exist because any firm can trigger search simply by offering a gain from search. All consumers would know that such a gain would exist, and be able to search to find the seller. In the case where (1) does not hold there is no pure strategy Nash equilibrium in the one shot game.[2] We therefore proceed by investigating mixed strategy equilibria.

Within a repeated game, a cartel may wish to maintain a monopoly or near monopoly outcome by utilising the possibility of punishment.. Then we have the following sequence of events:

- At the start of a period, the cartel chooses prices p^c for its members. These prices may not be all the same. If they are different, high and low prices will be randomly cycled among the firms so that each firm has the same expected price over future periods. All firms know all prices chosen by the cartel.

[1] This is exactly equivalent to a result of Shilony (1977)

[2] Consider n=2. Then if both prices were the same but less than m either firm would wish to increase its price up to m or to s above the other's price. Alternatively, assume the firms set different prices. If this difference is less than s the lower pricing firm would wish to increase its price, while if the difference was greater than s the higher pricing firm would wish to reduce its price to capture some sales. Hence the only possible Nash equilibrium in pure strategies in when both price at m, but this requires the condition (1).

- Each firm simultaneously sets its price at the level requested by the cartel, p^c , or at another (deviant) price $p^d \neq p^c$.
- Each consumer knows one price and can buy at that price with no further cost, and is told the minimum price available elsewhere. To buy at the latter price they need to spend a search cost s. Consumers make rational consumption decisions.
- At the end of the period, the cartel reviews actual prices set by its members and repeats the game, with a new allocation of the same set of prices, if no deviant prices are observed. It enters a punishment phase if any deviant price(s) are observed.

We now consider the nature of the punishment strategy needed to maintain the adherence of an individual firm to the cartel's pricing policy. It is natural to focus on this policy being to charge the monopoly price. However, a punishment strategy may still be needed in cases where the full monopoly price is not maintained by the cartel. In the standard manner, we structure the problem as one in which there is a one period gain from cheating, followed by a τ-periods punishment phase, in which the punishment is designed to dissuade firms from reneging on the agreement (choosing prices other than those allocated by the cartel). The value of τ represents a weighted sum of period values discounted to the present. In the simplest case, the temptation to an individual firm when the monopoly price is maintained by all others but (1) does not hold, is a one period gain equal to the revenue from selling N at price m-s-ε (where ε is arbitrarily small but positive) rather than selling N/n at price m:

$$Gain = N(m - s - \varepsilon - m / n) \qquad (2)$$

More generally the cartel may set a range of prices for its members, with $p_1 \geq p_2 \geq p_3 \geq ... \geq p_n$ and with mean price $\bar{p}$. For all members to receive positive sales we need and assume that $p_1 - p_n \le s$. The largest temptation is for a firm allocated price p_n. It is a one-period gain from selling $N(n'+1)/n$ at a price $p_{n'} - s - \varepsilon$, rather than selling N/n at price p_n, for any $n' < n$. The maximum temptation is the maximum value of (3) for all $n' < n$, at say n^*:

$$\text{Gain} = \max_{\{1 \le n' < n\}} \left(\frac{N}{n}((p_{n'} - s - \varepsilon)(n'+1) - p_n) \right) \qquad (3)$$

The punishment for cheating would be to forgo the average future profit ($\overline{p}N/n$) for the profit the deviant firm could still obtain, for the duration of the punishment phase. Let the punishment strategy take the following form. For a total of τ discounted periods the cartel allocates a price p_L to one of its (remaining) members and p_L+s to the others. The low price p_L limits the profit the cheating firm can obtain in this punishment phase. The cheating firm can do no better than to respond to punishment by setting a price of p_L+s, provided that:

$$(p_L - \varepsilon)N\frac{n-1}{n} < (p_L + s)\frac{N}{n} \qquad \text{(n>2)} \qquad (4)$$

and

$$(p_L - s - \varepsilon)N < (p_L + s)N/n \qquad (5)$$

i.e. we need $p_L - \varepsilon < s/(n-2)$ for (4) and $p_L - \varepsilon < \frac{n+1}{n-1}s$ for (5)

Constraint (4) states that the firm should not prefer to undercut all but the lowest price firm; constraint (5) states that the firm will not prefer to undercut all firms. The tighter of constraints (4) and (5) will hold, and therefore the conditions (in the limit as $\varepsilon \to 0$ [3]) under which the cheater will stick to p_L+s may be rewritten:

$p_L \le \frac{s}{n-2}$, if $n > 2$ since (4) is then the tighter constraint.

$p_L \le 3s$, if $n = 2$ since (5) is then tighter.

Then, whilst the punishment phase is proceeding, the cheating firm will achieve $(p_L + s)N/n$. Thus the loss due to punishment is

$$Loss = \tau(\overline{p}\frac{N}{n} - (p_L + s)\frac{N}{n}) = \tau\frac{N}{n}(\overline{p} - (p_L + s)) \tag{6}$$

For cheating behaviour to be non incentive-compatible, from (3) and (6) it is required that

$$(p_{n^*} - s)(n^*+1) - p_n < \tau\left(\overline{p} - (p_L + s)\right)$$

$$\text{or} \quad p_L < \left(\tau\overline{p} + p_n - p_{n^*}(n^*+1) + s(n^*+1-\tau)\right)/\tau \tag{7}$$

Clearly (7) holds when all prices are monopoly prices *m* in circumstances where (1) holds since the gain is then negative when $p_{n^*} = p_n = m$.

If (1) does not hold, it may still be the case that the relative effective punishment length (given by τ) and depth (given by p_L) are sufficient to enable (7) to hold when the cartel sets all prices as monopoly prices: hence monopoly prices could be maintained by the cartel. Otherwise the cartel may be able to maintain only prices some or all of which are less than *m*. In these cases, the relevant frontier for p_L is the highest p_L such that the temptation is no more than the punishment:

$$p_L = \frac{1}{\tau}\left(\tau\overline{p} + p_n - p_{n^*}(n^*+1) + s(n^*+1-\tau)\right) \tag{8}$$

and the (*n*-1)-firm cartel's total loss from carrying out the punishment relative to monopoly pricing is:

$$TL = N\left[(n-1)(\overline{p} - p_L)/n - (n-2)s/n\right]\tau \tag{9}$$

[3] To avoid cluttering the equations, we henceforth take ε as its limiting value of zero.

Substituting (8) into (9), it is easily seen, as expected, that the total loss is minimised by making τ as small as possible (by choosing the lowest punishment length) given that p_L is non-negative.[4] This lower level is bounded by the requirement that (8) is satisfied at p_L=0, whence

$$\tau = \frac{-p_n + p_{n*}(n*+1) - s(n*+1)}{\overline{p} - s} \qquad (10)$$

If this value of τ is achievable at $\overline{p} = p_n = p_{n*} = m$, (when the required τ is equal to n-$m/(m$-$s)$) then an equilibrium in which all firms price at the monopoly level may be maintained. This equilibrium, unlike the Diamond case where (1) holds, requires at least tacit co-ordination amongst the firms to maintain a punishment strategy that will be employed in the event of a cheat occurring.

The achievable τ might in practice be bounded somewhat below $n - m/(m - s)$, particularly if n is large or s is small. For example, the future market may be liable to intervention by entrants, regulators or technological innovation. Any of these factors may reduce the likelihood of profitability continuing for many periods. In such a case, although the cartel cannot support full monopoly pricing, co-ordination coupled with a punishment strategy can assure prices with an average that, whilst lower than m, is higher than that in a competitive case. In order to investigate this possibility in more detail, and particularly to consider the question of how consumer search costs interact with the punishment strategy, we move to a special case with three firms. This allows us to capture all the essential insights without unnecessary complication.

[4] The resulting expression after substitution is linear in τ, and the coefficient of τ is s/(n-1).

3. A Three Firm Case

Consider an industry of three firms, with zero production costs. Consumer demand and search technology are as before. The cartel wishes to maximise its (average) revenue subject to constraints. Without loss of generality, the firms set prices p_1, p_2, p_3 that may be different from each other, price p_1 being weakly highest and p_3 lowest. However, these will be randomised so that, over time, each firm receives the average price. Clearly, no firm will set a price greater than m. Also, the difference between prices p_1 and p_3 must not exceed s in equilibrium, in order that no consumer has an incentive to shift supplier. In the absence of these constraints, p_1 would lose both sales and relevance. Finally, there are two incentive compatibility constraints and we discuss these below.

The temptation to cheat is strongest for the firm allocated the lowest price, p_3. From (3), it has two possible ways of cheating - by setting a price more than s below p_2 and hence capturing the entire market, or by setting a price more than s below p_1 thus obtaining 2/3 of the market. The one-period gain is the difference between the profit obtained from cheating and the profit from firm 3's "fair" share of the market, which is $N\, p_3/3$. Cheating will be deterred if the effect of the punishment is greater than the one period gain. The toughest punishment is where, for a discounted sum of periods τ, one of the other two firms set price at zero, meaning the cheater can obtain only $Ns/3$ rather than the long run expectation $N\,\overline{p}\,/\,3$, where $\overline{p}$ is average price.

Therefore, setting out the cartel's problem, it wants to maximise group revenue subject to the various constraints:

Max $N\,\overline{p} = N(p_1 + p_2 + p_3)/3$ with respect to p_1 , p_2 , p_3

Subject to

$p_1 \geq p_2 \geq p_3 \geq 0$ P0

$(p_2 - s)3/3 - p_3/3 \leq \tau[(p_1 + p_2 + p_3)/9 - s/3]$ P1

$(p_1 - s)2/3 - p_3/3 \leq \tau[(p_1 + p_2 + p_3)/9 - s/3]$ P2

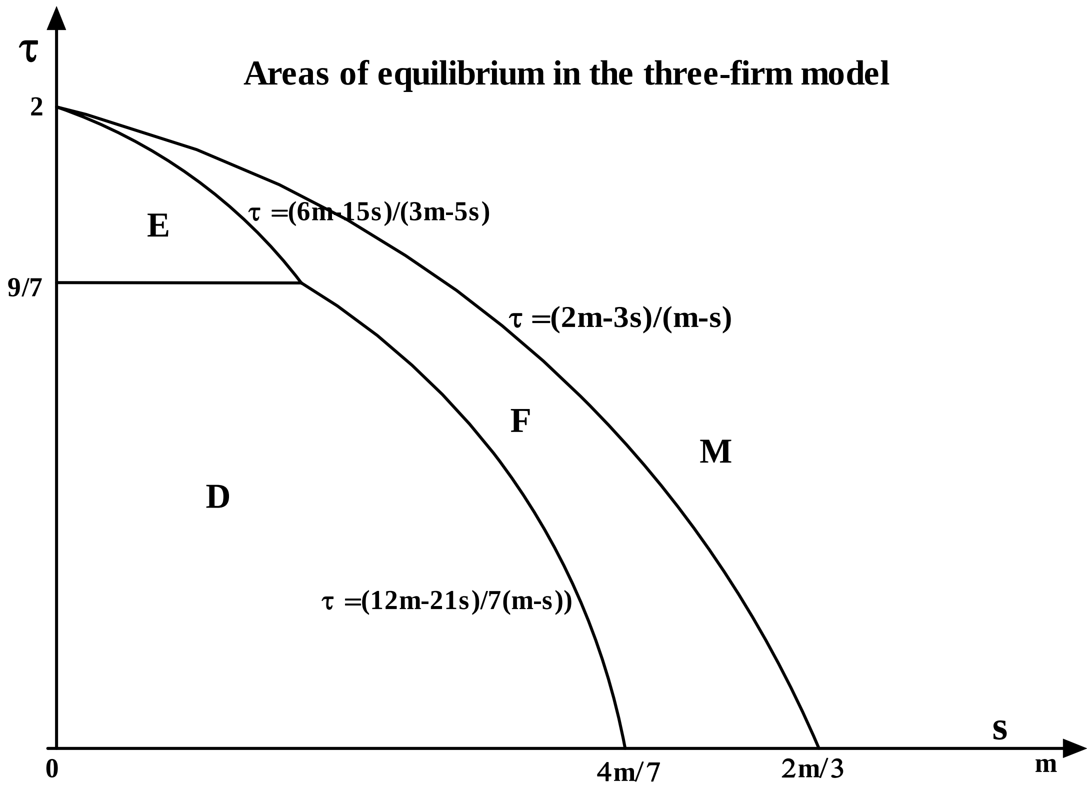
$p_1 - p_3 \leq s$ P3

$p_1 \leq m$ P4

Immediately we note that, for any choice of p_1, p_2, all constraints (apart from the first) are relaxed and the objective function is increased if p_3 is set at p_2. The intuition here is of interest. Setting the lowest price less than the next lowest reduces the average price gained but also increases the difficulty of stopping the firm allocated the lowest price from cheating. Thus we know that p_3=p_2 and the problem becomes one of maximisation of $(p_1 + 2p_2)N/3$ with respect to p_1 and p_2 with at most two constraints binding (other than P0). This problem may be solved as a linear programming problem and the solution is set out at length in the Appendices.

The various outcomes are best seen in terms of a diagram, as in figure 1 below. In this diagram the equilibrium type is shown in terms of the parameters s and τ. First, the locus along the s axis from $2m/3$ to m is the equivalent in our framework of the Diamond result. The area above that marked M is the region for which a cartel can maintain all prices at the monopoly level of m through a suitable punishment. Note this assumes a high enough value of τ is available for any value of s - if this is not so, collusion cannot be sustained. Given the search technology assumed, this area clearly shows that the higher are search costs, the lesser need be the punishment required (in the sense of lower τ). Hence the area M is characterised by full monopoly pricing enabled by either punishments ($\tau > 2$), or search costs (s > 2m/3) or a combination of punishment and search costs. Only constraints P0 and P4 are binding.

Figure 1



Type M: τ, s are such that all firms in the cartel can set monopoly price of m.
Type D: no firm sets monopoly price and there is no consumer search if cartel members do not cheat. Both high and low prices set to stop cheating.
Type E: lowest price is set to stop cheating to steal all markets; highest price is set less than m to stop consumers from searching and avoiding highest price firm.
Type F: Highest price is set at monopoly price of m; lowest price set to stop cheating to steal all markets.

For any given value of τ, as s falls, the next region reached is the one marked F. Here the top price remains at m, but the other two firms set a price below this given by

$$p_2 = \frac{9s - 3\tau s + \delta m}{6 - 2\tau}.$$

Constraints P1 and P4 are binding at these prices. Thus if p_2 were increased towards the monopoly price it would pay firm 3 to undercut both other firms by s and break the cartel strategy.

At still lower values for s, region D is entered (assuming τ is below 9/7). In this region, we have

$$p_1 = \frac{7s(3-\tau)}{12-7\tau} \text{ and } p_2 = \frac{s(18-7\tau)}{12-7\tau}.$$

Here both prices have to be reduced below m since it would otherwise pay firm 3 to undercut firm 1 by setting a price of m-s, even though this would only gain one firm's customers. In this equilibrium, both P1 and P2 are binding.

Alternatively, in region E,

$$p_1 = \frac{s(15-5\tau)}{6-3\tau} \text{ and } p_2 = \frac{s(9-2\tau)}{6-3\tau}.$$

Here, there is a different reason for the lower p_1. If p_1 is not reduced from m to this value, then buyers will switch from firm 1 to the other firms. In this equilibrium P1 and P3 are binding.

In both D and E equilibria we see the same effect. As s falls, both prices are forced below m unless a τ value above 2 can be sustained. The behaviour of prices, and hence of average prices (cartel revenue), is continuous across all regimes. Both high and low prices are monotonically non-decreasing with respect to both s and τ.

To summarise, region F is "near" region M. Region D is based on preventing cheating by acquiring both the other firms' customers or by acquiring just customers at the highest price firm. Region E is based on preventing cheating for the whole market, but the price range is constrained by the threat of consumers' search. Region E reflects relatively low search costs and high punishment capability. Region D reflects relatively high search costs and low punishment capability.

Several novel features of this set of equilibria are worth pointing out. First, as already said, it may be that a cartel can sustain different prices rather than having each firm set the same price. In this case, it is to be expected that the identity of the low price firm will change frequently. This is a phenomenon that is consistent with observations in settings where search costs are low, such as for internet purchases, rather against the early expectations regarding this market. Of course we may note that price differences may rather reflect a number of other possibilities, either that the industry is in a mixed strategy equilibrium, or that the industry is in disequilibrium, or that a form of collusion is being maintained that does not require punishments in order to be sustainable. To see the last of these, consider the case in region D as τ tends to zero, so no punishment can be enforced. Then the limiting values of p_1 and p_2 become $p_1 = 7s/4$ and $p_2 = 3s/2$ in this case, from maximising average price while applying constraints P1 and P2.

The current setting also brings into question the investigation of and policy towards collusion. Here by construction, throughout the diagram, collusion is taking place, although it would not necessarily be evidenced by parallel prices. Also, the consequences for consumers differ greatly. Thus some collusions are "more serious" than others in terms of their effects on prices. Generally, collusion resulting in differential prices is less serious in its consequences for consumers than collusion with equal prices. Also, sometimes collusion is taking place by means of punishment threats and sometimes by coordination, using the imperfect consumers' information as the basis for monopoly.

4. The introduction of more efficient search technologies

The framework developed so far relies on all consumers having the same level of search cost. We now consider what happens if a subset of consumers has a lower search cost. We may imagine that some consumers adopt a new technology of search (say, through the internet). Under what circumstances will this disturb the equilibrium? This question is considered below only for the three firm case in which $\tau = 0$, i.e. along the *x*-axis of Figure 1. For convenience, we set $m = 1$.

In the existing equilibrium, $(p_1, p_2, p_3) = (7s/4, 3s/2, 3s/2)$. Recall that the search technology involves consumers knowing the price of the firm at which they have seen the product, plus the lowest firm's price, but not the location of the cheapest price. Customers can pay s to obtain this information. There are N existing consumers. The new set of consumers is of measure μN (and we suppose $\mu < 1$). They enjoy a lower search cost $s_I < s$. Firms, by assumption, cannot distinguish such customers ex ante, and therefore cannot discriminate in price between end user types.

The existing price vector will fail as an equilibrium if the profit obtained from a reduced price is higher than the profit from maintaining the existing equilibrium price.[5] Note that there are two ways in which the equilibrium might be broken. First, (the player playing the role of) firm 1 may lower price towards or below the other prices in response to the set of new customers seeing it as high price and therefore taking their custom elsewhere. A necessary condition for this to happen is that $s_I < s/4$, since otherwise the reported price gap is insufficient to encourage search. Second, one of the players may reduce price below either existing price in order to capture all the new customers. In this case, firm 3 has a (weakly) greater incentive to reduce price since its profits currently are (weakly) lowest. This could happen in principle for any value of s_I and dominates the first possibility.

Consider then the decision of firm 3. Its profit this period, assuming it maintains the existing equilibrium, is: $(N/3)(1+\mu)(3s/2)$. If it cut price sufficiently to attract the new consumers to search, that is a cut to a level at least s_I below p_2, then all the new customers would come to its store. In this case, its profit this period is: $N(1/3+\mu)(3s/2-s_I)$.

Hence a necessary condition for the existing equilibrium to be broken is:

$$(1/3+\mu)(3s/2-s_I) > (1/3)(1+\mu)(3s/2)$$

Simplifying, this condition becomes:

[5] Note that by choice of example, no punishment is required here.

$$\mu > \frac{s_I}{3(s - s_I)} \tag{10}$$

For example if $s_I = 0.5s$, μ > 1/3 is necessary; if $s_I = 0.1s$, only μ >1/21 is needed.

In conclusion, the existing equilibrium is immune to the entry of *small* numbers of consumers who search on the basis of lower search costs than the majority. However, the larger the search cost reduction these consumers enjoy, the smaller in size they need to be as a group in order not to disturb the existing equilibrium.

5. The Concrete Case

Our model permits the analysis of a number of policy and strategic issues. Any reduction in either s or τ will reduce at least one price unless the initial position is within the interior of M and the policy effect is too small to shift to another region of equilibrium. Thus we can view any reduction in either of these parameters as beneficial to consumers. Given that total surplus is fixed in the absence of any actual search expenditure, a gain for consumers is a loss for the firms and vice-versa. A cartel might well seek to increase search costs. On the other hand, a policy of seeking to reduce search costs is one that can be pursued by government, regulators or consumer organisations. The simplest instrument through which to reduce search costs is to require transaction prices to be public, either as posted prices to which firms are required to maintain, or as ex post reports on prices paid. The issue arises as to whether such a move will also affect τ in a way that is adverse to consumers. For example, τ is the (discounted) weight of the number of time periods for punishment relative to the length of time that cheating can persist in the absence of a cartel response. If transparent prices lead to faster detection of cheating then the time before detection becomes shorter relative to the punishment phase: hence we can view this as τ increasing. If τ increases at the same time as s decreases as a result of a policy to make pricing more transparent, then the direction of movement in the diagram is to the north-west and the net effect is ambiguous.

Albaek et al (1997) argue that this issue is a real problem. They discuss a move by the Danish anti-trust authority to make prices of key ready-mixed concrete products more transparent by surveying invoices and publishing prices. They find that this reform was accompanied by increases in invoice prices. Their argument is that previous discounts were phased out due to the difficulty of keeping information about these from other suppliers. Thus the transparency which was thought to aid consumers (reduced s in our model) actually led to more collusion (higher τ in our model) and higher actual prices. The paper makes its argument purely from the empirical facts, concentrating on rejecting other reasons why prices might have risen at this time. A further paper (Overgaard and Mollgaard, 2005) considers this issue at more length, addressing the relevance of folk theorems to sustainable collusion. However, it does not address the question of why non-uniform prices could be observed in markets where cartel and search issues are present.

Our analysis can be extended to show that the policy effect on average prices could go either way. Again we assume m = 1. We suppose a policy instrument is available which changes s by ds, but also changes τ by $d\tau = -\theta ds$. We take $\theta>0$ since otherwise there is no question as to the optimal direction of policy. We will see that there is an upper bound for θ in each of the equilibrium regions D, E and F such that the policy change involving a reduction in s ($ds<0$) reduces average price. For each of these types of equilibrium, we have average price and the upper bound on θ as

Region D:

$$p_1 = \frac{7s(3-\tau)}{12-7\tau} \text{ and } p_2 = \frac{s(18-7\tau)}{12-7\tau}.$$

Therefore

$$\bar{p} = (p_1 + 2p_2)/3 = (19 - 7\tau)s/(12 - 7\tau)$$

and

$$d\bar{p}/ds = (19 - 7\tau)/(12 - 7\tau) - \theta 49s/(12 - 7\tau)^2 > 0 \text{ iff}$$

$$\theta < (19 - 7\tau)(12 - 7\tau)/(49s) \equiv \theta^D$$

Region E:

$$p_1 = \frac{s(15-5\tau)}{6-3\tau} \text{ and } p_2 = \frac{s(9-2\tau)}{6-3\tau}.$$

Therefore

$$\bar{p} = (p_1 + 2p_2)/3 = (11 - 3\tau)s/(6-3\tau)$$

and

$$d\bar{p}/ds = (11 - 3\tau)/(6 - 3\tau) - \theta 15s/(6 - 3\tau)^2 > 0 \text{ iff}$$

$$\theta < (11 - 3\tau)(2 - \tau)/(5s) \equiv \theta^E$$

Region F:

$$p_1 = m \qquad p_2 = \frac{9s - 3\tau s + \tau m}{6-2\tau}.$$

Therefore

$$\overline{p} = (p_1 + 2p_2)/3 = s + m/(3-\tau)$$

and

$$d\,\overline{p}\,/ds = 1 - \theta m\,/(3 - \tau)^2 > 0 \text{ iff}$$

$$\theta < (3 - \tau)^2/m \equiv \theta^F$$

Thus in any of the equilibrium regions where prices are affected by a small change in s (and opposite sign change in τ) the possibility of moving to a worse average price exists. Even when this is not going to occur it is possible that some consumers will benefit and others will lose in any particular period. To see this, suppose we are in region D with s=0.2 and $\tau = 0.5$. Then suppose we take a small finite change with ds = -.01 and dτ = .12, so that $\theta = 12$. At the initial point θ^D is 13.44. Here, p_1= 0.412 and p_2=p_3= 0.341. After the change, we have p_1= 0.413 and p_2=p_3= .339. In this case the joint effect of the policy change has been to increase the high price and decrease the low prices (so the range of prices has increased) while average price has decreased.

As an alternative to marginal policy changes considered above we could also look at large shifts in market parameters. For example, consider a move towards complete consumer information and hence s=0. Such a move would take the equilibrium to the τ axis in Figure 1. Here all points below $\tau = 2$ have all prices equal to 0. Above $\tau = 2$ all prices are set at the monopoly level of m. Thus the policy reform is in the consumers' interest unless it changes τ from below 2 to above 2.

6. State-Specific Strategy

The analysis above reflects the constant-sum benefits of consumer surplus and profit from our simple model. Just as the Government might wish to minimise prices so as to maximise consumer surplus, so the cartel wishes to maximise prices to maximise profits of its members. The four states represented by the four regions M, D, E, and F in figure 1 show that the amount of change in prices from some policy affecting s and τ is dependent on the initial location and whether the change jumps regions. The government will generally wish to reduce s and τ while the cartel wishes to increase both. The comments relating to government policy in the Concrete Case above can thus be simply inverted and transferred to the issue of cartel strategy, and no further discussion is needed.

However, the instruments that the cartel can use as part of its strategy may have other or additional implications, and these may also be state-specific. For example, the cartel may consider advertising its products at some joint cost A, emphasising the quality of its products to consumers. Alternatively the cartel could actually enhance the quality of its product. In either case, this could be thought to raise m to $m'=m+\Delta$.[6] The distribution of benefits depends radically on the location of the initial equilibrium. If the equilibrium is in region M then all the extra value (perceived or real) of the product can be extracted by the cartel as profit and there is no consumer surplus. In regions D and E there is no change to any price. Thus all the extra value accrues to consumers and there is no extra profit. In region F, p_1 increases to m' and

[6] See e.g. Johnson and Myatt (2006) for a discussion of the possible impacts of advertising on consumers.

p_2 to $\frac{9s-3\tau s+\tau m'}{6-2\tau}$. Here therefore the increase in value is shared. Profits increase by $(1+2\tau/(6-2\tau))N\Delta/3$ while consumer surplus increases by $2(1-\tau/(6-2\tau))N\Delta/3$. Thus the expenditure of A to obtain Δ, which is to the cartel's advantage if the cartel is in the M region, may still be advantageous in region F, but will not be to the cartel's advantage in the E or D regions. Only in Region F might consumers and cartel both strictly gain.

7. Entry

We have been considering a cartel of all the firms in the market. Since the cartel operates explicitly or implicitly in the interests of all its members, an entrant into the market would find it advantageous to join the cartel (otherwise it would be punished as a deviant). Complications include the possibility that more firms imply a smaller basic number of consumers for each firm, since this might lead to punishing entry as such, or that more firms imply that the temptation to cheat was larger. First, entry into this market may have no significant impact on other firms' basic numbers of consumers. This would be the case if the new firm adopted a location away from others and so most of the consumers who first see the product at this new location would not have seen it at any other location. Hence a new entrant expands the market size N, perhaps even proportionately. If N/n is unchanged with entry and p_L is already zero (see (4)), then the loss due to punishment (see (3)) is unchanged while the temptation is increased by the cheating firm being able to multiply sales, by (n+1) rather than n, by reducing price to s below m. Thus the cartel becomes less able to enforce monopoly prices as it becomes larger due to entry and hence may require a longer punishment period to enforce monopoly prices, or the cartel may have to settle for lower average prices.

8. Concluding remarks

The framework employed in this paper is deliberately simple in order to make the points in the clearest way possible. It has enabled us to demonstrate a link between consumer search and cartel behaviour, bringing together two separate literatures. One explains consumer search and the incentive for consumers to search (or not) provided by price variance among firms. The other considers the impact of possible consumer search on the temptation of individual cartel members to renege on the (implicit or explicit) cartel agreement, as well as the discipline issues that result. By unifying these two areas of analysis we have revealed possibilities for equilibrium cartel behaviour, in particular for the existence of different prices across firms, which have not previously been considered. It is also noteworthy that a marginal increase in the number of low-search-cost consumers need not destroy the character of the existing equilibrium outcome, although at some point the effect will be significant. Moreover the model has allowed us to assess the impact of policies designed to increase market transparency in a far more complete way than hitherto.

It is often argued that the key policy issue relating to cartels is the evidence that a regulatory body can find to indicate that a cartel exists.[7] For example it might be proposed that whilst low variance in prices across competing firms coupled with low profit levels does indeed suggest active competition, low variance in prices and *high* profitability is indicative of (implicit or explicit) cartelisation. Our equilibria demonstrate the first possibility in the case where both τ and s are very small. Here

[7] See e.g. Harrington (2005) and Grout and Sonderegger (2005) for useful discussions.

the consumer searches to find the lowest price and there is no power to discipline individual firms that determine to undercut others. By contrast, our equilibrium M would be categorised as requiring investigation for anti-trust behaviour on the grounds that it fits with the second set of observations.

However we must first recognise that an equilibrium M arises from a combination of search costs and cartel punishments. The implication of the balance between the two factors is important. The policy response from anti-trust authorities to high search costs should be very different from that relating to direct cartelisation. High search costs would prompt a policy of improved consumer information, whereas cartel maintenance activity implies warnings about or actions related to correlated activities.

A second and novel policy issue relates to the identification of cartelisation when there is some variance of prices across suppliers. Our equilibria of types D,E and F are all characterised by cartel members setting different prices. On the one hand there is the desire to reduce price differences to eliminate the incentive for consumers to search; on the other there is the need to restrict the temptation of an individual firm to cheat on the cartel. Hence in equilibrium there may be a limit to the prices that the cartel can sustain without incurring cheating and this can be addressed by limited price variation so that it is not possible to "just undercut" all other firms. Also there is a limit to such preventative methods since the cartel wishes to avoid additional consumer search. These equilibria, involving a cartel maximising its constrained aggregate profit, imply that price variation may be a feature of a cartel which lacks the disciplinary power to set full monopoly prices, given consumers' search costs. The policy response from an anti-trust authority should be both to improve consumer information and to investigate whether profits are excessive. The key point is that observed price variations across players do not guarantee that the competitive mechanism is working in a satisfactory way.

Similarly, price variation in a firm's prices across time is not necessarily indicative of active competition. Since firms in the cartel which sets non-uniform prices have different profits, their roles would change from one period to another to ensure that cartel breakdown has the same expected punishment to all firms. This price volatility is thus part of an equilibrium cartelisation rather than indicating active competition. Intertemporal price variations have the effect of ensuring that consumers do not learn the location of high or low prices from past experience, and this is an integral feature of our model.

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Appendix: the Solution to the Linear Programming Problem

A1 Simplifying the Cartel's Problem

The cartel's problem is to maximize the cartel's revenue subject to incentive compatibility constraints and the bound on maximum price.

Max $N\overline{p} = (p_1 + p_2 + p_3)N/3$ with respect to p_1 , p_2 , p_3

Subject to

$p_1 \geq p_2 \geq p_3 \geq 0$ P0

$N[(p_2 - s)3/3 - p_3/3] \leq N\tau[(p_1 + p_2 + p_3)/9 - s/3]$ P1

$N[(p_1 - s)2/3 - p_3/3] \leq N\tau[(p_1 + p_2 + p_3)/9 - s/3]$ P2

$p_1 - p_3 \leq s$ P3

$p_1 \leq m$ P4

For any choice of p_1, p_2, all constraints (apart from the first) are relaxed and the objective function is increased if p_3 is set at p_2. Therefore replace p_3 by p_2 and change variables in the following way.

$x_1 = p_1 - p_2$; $x_2 = p_2 - s = p_3 - s$

An equivalent problem can be found by substituting x_1 and x_2 for p_1 and $p_2 = p_3$ into the objective function and constraints of the original problem. First note that $\overline{p} = (x_1 + 3x_2)/3 + s$. Thus maximizing $N\overline{p}$ is equivalent to maximizing $Z = x_1 + 3x_2$. We just have to remember that $\overline{p} = Z/3 + s$ and $N\overline{p} = N(Z/3 + s)$. The constraints are rewritten in terms of the x variables, and the equivalent problem is

Max $Z = x_1 + 3x_2$ with respect to x_1, x_2

Subject to

$-\tau/3\ x_1 + (2-\tau)x_2$	$\leq s$	P1
$(2 - \tau/3)\ x_1 + (1-\tau)\ x_2$	$\leq s$	P2
x_1	$\leq s$	P3
$x_1 + x_2$	$\leq m\text{-}s$	P4
$x_1, x_2 \geq 0$		(P0)

The non-negativity constraints $x_1, x_2 \geq 0$ are satisfied as part of the solution routine for linear programming. The equivalent problem is thus a standard LP problem with two variables and 4 linear inequality constraints.

A2 Solving the Equivalent Problem

Consider the following dual linear programming problems.

Primal

Max $Z = x_1 + 3x_2$ with respect to x_1, x_2

Subject to

$-\tau/3\ x_1 + (2-\tau)x_2$	$\leq s$	P1
$(2 - \tau/3)\ x_1 + (1-\tau)\ x_2$	$\leq s$	P2
x_1	$\leq s$	P3
$x_1 + x_2$	$\leq m\text{-}s$	P4
$x_1, x_2 \geq 0$		

Dual

Min $L = su_1 + su_2 + su_3 + (m\text{-}s)u_4$ with respect to u_1 u_2 u_3 and u_4

Subject to

$-\tau/3\ u_1 + (2 - \tau/3)\ u_2 + u_3 + u_4 \geq 1$ D1

$(2\text{-}\tau)u_1 + (1\text{-}\tau)\ u_2 + u_4 \geq 3$ D2

$u_1, u_2, u_3, u_4 \geq 0$

We can look for solutions where the primal problem has only one constraint binding and where it has two constraints binding. Nothing is added to the problem by considering cases where more than two constraints are strictly binding since a basis for the dual only involves two non-zero variables at most. We will see that we can identify optimal solutions for all feasible values of the parameters τ and s.

We will make use of the duality theorem[8] which says that feasible solutions to both problems, plus complementary slackness, constitute optimal solutions to both problems.

Solution with just one primal constraint binding.

If P1 is binding but others are not, then u_1 is the only non-zero dual variable, and D1 cannot hold. Thus **no solution** of this type exists.

[8] K Lancaster (1968, p29) states the duality theorem where the primal has a maximand of cx and the dual a minimand of yb as “A feasible vector x* of the primal problem is optimal, if and only if the dual has a feasible vector y* such that cx* = y*b. The vector y* is then optimal for the dual.” Furthermore,

If P2 is binding but others are not, then u_2 is the only non-zero dual variable, and D2 cannot hold if $\tau>1$ or $\tau=1$. If $\tau < 1$, $u_2 = \max\{3/(6-\tau), 3/(1-\tau)\} = 3/(1-\tau)$ and D2 is binding. But then $x_2 = 1/(1-\tau)$ from P2 and $x_1=0$ from D1 not being binding. Then P1 is not satisfied. Thus **no solution** of this type exists.

If P3 is binding but others are not, then u_3 is the only non-zero dual variable, and D2 is not satisfied. Thus **no solution** of this type exists.

If P4 is binding but others are not, then u_4 is the only non-zero dual variable. Solution to the dual is $u_4 = 3$, and D1 is not binding. Thus $x_1=0$ and $x_2= m-s$. This is a feasible solution to the primal problem if $(2-\tau)(m-s) \leq s$, or $s \geq (2-\tau)m/(3-\tau)$. Equivalently $\tau \geq (2m-3s)/(m-s)$.

We will term this **solution (M).** The set of parameter values yielding this solution is shown on figure 1.

Solution with two primal constraints binding

There are 6 possibilities.

(A) Only P3, P4 binding; then only u_3 and u_4 are non-zero. But then D1 is not binding and so $x_1=0$. Then P3 cannot be binding, and **no solution** of this type exists.

(B) Only P2, P4 binding; then only u_2 and u_4 are non-zero. Suppose that both x_1 and x_2 are non-zero in the primal solution. Then both D1 and D2 are equalities. Subtract D1 from D2 and obtain $u_2 = -6/(3+2\tau) < 0$ and is infeasible. If only one of D1 and D2

Lancaster also states the existence theorem as "The primal and dual have optimal solutions , if and only if both have feasible vectors."

are binding then only one of u_2 or u_4 would be positive and hence both P2 and P4 could not be binding. **No solution** of this type exists.

(C) Only P2 , P3 binding; then only u_2 and u_3 are non-zero. Suppose that both x_1 and x_2 are non-zero in the primal solution. Then both D1 and D2 are equalities. Subtract D1 from D2 and obtain $-(1+2\tau/3)u_2 - u_3 = 2$, which implies infeasibility. If only one of D1 and D2 is binding then only one of u_2 or u_3 would be non-zero and hence both P2 and P3 could not be binding. **No solution** of this type exists.

(D) Only P1, P2 binding; then only u_1 and u_2 are non-zero. Dual solution from D1 and D2 as equalities is

$u_1 = 15/(12\text{-}7\tau)$ $u_2 = 6/(12\text{-}7\tau)$ $u_3\text{=}u_4\text{=}0$ $L = 21s/(12\text{-}7\tau)$

Primal solution from P1 and P2 as equalities is

$x_1 = 3s/(12\text{-}7\tau)$ $x_2 = 6s/(12\text{-}7\tau)$ $Z = 21s/(12\text{-}7\tau)$

These solutions are feasible provided P3 and P4 are satisfied:

$3s/(12\text{-}7\tau) \leq s$ or $\tau \leq 9/7$ (P3)

$9s/(12\text{-}7\tau) \leq m\text{-}s$ or $\tau \leq (12m\text{-}21s)/(7(m\text{-}s))$ (P4)

(E) Only P1, P3 binding; then only u_1 and u_3 are non-zero. Dual solution from D1 and D2 as equalities is

$u_1 = 3/(2\text{-}\tau)$ $u_3 = 2/(2\text{-}\tau)$ $u_2\text{=}u_4\text{=}0$ $L = 5s/(2\text{-}\tau)$

Primal solution from P1 and P3 as equalities is

$x_1\text{=}s$ $x_2 = (3+\tau)s/(3(2\text{-}\tau))$ $Z = 5s/(2\text{-}\tau)$

and is a feasible solution if and only if P2 and P4 are satisfied. For this we require

$(2-\tau/3)s + (1-\tau)(3+\tau)s/(3(2-\tau)) \leq s$ or $\tau \geq 9/7$ (P2)

$s + (3+\tau)s/(3(2-\tau)) \leq m-s$ or $(3m-5s)\tau \leq (6m-15s)$ (P4)

(F) Only P1, P4 binding; then only u_1 and u_4 are non-zero. Dual solution from D1 and D2 as equalities is

$u_1 = 3/(3-\tau)$ $u_4 = 3/(3-\tau)$ $u_2=u_3=0$ $L = 3\mathrm{m}/(3-\tau)$

Primal solution from P1 and P4 as equalities is

$x_1 = [3m(2-\tau) - 3s(3-\tau)]/(6-2\tau)$ $x_2 = [\tau m + (3-\tau)s]/(6-2\tau)$ $Z = 3m/(3-\tau)$

and is a feasible solution if and only if P2 and P3 are satisfied and $x_1 \geq 0$. For this we require

$(2-\tau/3)\,[3m(2-\tau) - 3s(3-\tau)]/(6-2\tau) + (1-\tau)[\tau m + (3-\tau)s]/(6-2\tau) \leq s$

or $\tau \geq (12m-21s)/(7(m-s))$ (P2)

$[3m(2-\tau) - 3s(3-\tau)]/(6-2\tau) \leq s$ or $(3m-5s)\tau \geq (6m-15s)$ (P3)

$[3m(2-\tau) - 3s(3-\tau)]/(6-2\tau) \geq 0$ or $\tau \leq (2m-3s)/(m-s)$ for $x_1 \geq 0$.

A3 Derivation of Figure 1

The four regions in Figure 1 are obtained by plotting the 4 critical boundaries:

$\tau = (2m-3s)/(m-s)$. (above this the monopoly solution, M is supported; below F is supported)

$(3m-5s)\tau = (6m-15s)$ (divides (E) (permitted below) and (F) (permitted above))

$\tau = (12m\text{-}21s)/(7(m\text{-}s))$ (divides (D) (permitted below) and (F) (permitted above))

$\tau = 9/7$ (divides (D) (permitted below) and (E) (permitted above))